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THE STUDY OF MASS TRANSFER FROM THE WALL OF A SMOOTH TUBE TO A TURBULENT LIQUID FLOW AT HIGH SCHMIDT NUMBERS

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Аннотация—В работе приводятся экспериментальные данные по массоотдаче от стенки гладкой трубы к турбулентному потоку жидкости в интервале чисел Прандтля от 1000 до 31000 и чисел Рейнольдса от 10000 до 60000.

Полученные данные подчиняются соотношению $Nu \sim Pr^{0,5}$.

NOMENCLATURE

λ, wavelength;
K, mass-transfer coefficient;
m, amount of substance dissolved;
t, time;
ΔC_m, $= \dfrac{(C_{sat} - C_i) + (C_{sat} - C_f)}{2}$ mean driving force;
C_{sat}, substance solubility;
C_i, initial concentration;
C_f, finite concentration;
S, inner surface of test section;
D, molecular diffusion coefficient;
ν, kinematic viscosity;
ω, rotational velocity of disc;
Nu, $= \dfrac{Kd}{D}$;
d, diameter of pipe;
$Pr(Sc)$, $= \dfrac{\nu}{D}$;
Re, $= \dfrac{vd}{\nu}$;
V, liquid flow velocity;
ζ, resistance coefficient.

To develop the theory of turbulent transfer in a viscous sublayer, sufficiently accurate experimental data on the rate of solution of solids in liquids over a wide range of Prandtl number (Schmidt number) are urgently needed.

The aim of the present work is to study the kinetics of solution from the inner surface of a cylindrical tube to a turbulent liquid flow. The experimental data which are available in the technical literature require additional verification [1] and cover a relatively narrow range of Prandtl number (up to 4000). In the study carried out by the authors the range of Schmidt number was extended up to 31000.

The experimental apparatus is shown schematically in Fig. 1. The circulation of the solvent in the system was performed by means of the rotary pump 1. The diameter of the vertical portion of the pipe 4 was 50 mm, and of the horizontal one, 13 mm. The length of the horizontal portion preceding the test section 8 was 100 times the diameter, which ensured the stabilization of the flow. The temperature in the working tank 12 was kept constant by means of a coil heat exchanger; the blade mixer 10 produced a uniform distribution of the solute in the tank. To regulate and control the liquid flow rate the control valve 5 and diaphragm 3 were mounted on the vertical portion of the pipe. For calibrating the flowmeter the measuring tank 13 was used; it could be filled with the solvent by turning the tube 9,

the liquid level being kept constant in the tank 12.

The test section of the pipe (Fig. 2) was a hollow cylinder 197 mm long with three connections, the centre one 4 was designed for

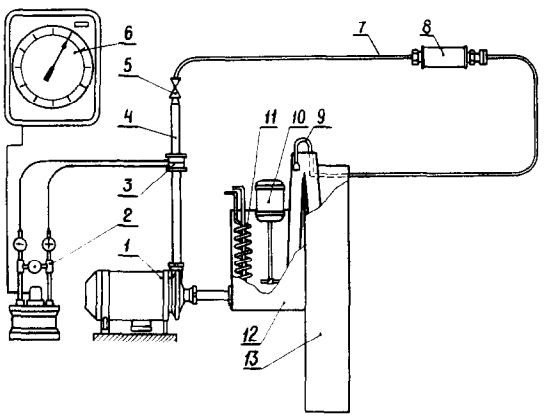

FIG. 1. Diagram of apparatus. 1—rotary pump; 2—differential manometer; 3—diaphragm; 4—vertical portion of delivery pipe; 5—control valve; 6—ЭПИД-20; 7—horizontal portion of pipe; 8—test section; 9—diverter pipe; 10—mixer; 11—heat exchanger; 12—working tank; 13—measuring tank.

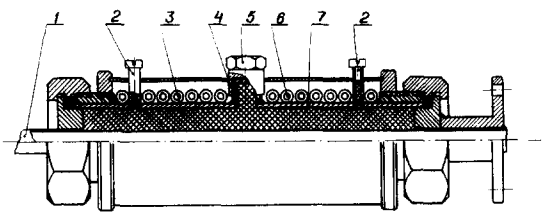

FIG. 2. Test section of pipe. 1—horizontal pipe; 2—connection for air release; 3—substance under investigation; 4—connection for pouring; 5—plug; 6—electric heaters; 7—body of working section.

pouring the studied material melt, and the side ones 2, for permitting air to flow from the section. Electric heaters 6 were set into the body of the test section.

Before pouring a carefully polished rod made of stainless steel was inserted into the test section. The outer diameter of the rod was equal to the inner one of the pipe 1. The rod entered the pipe through a sliding fit, which prevented the melt from splashing into the pipe during pouring. A supply funnel with electric heaters was screwed on the pouring connection. In this funnel the substance under investigation was melted. The melt flowed into the test section which was also heated beforehand up to the melting point of the material being studied. Cooling of the melt was performed very slowly to avoid cracks on the surface. From the cooled surface the rod was extracted by means of a screw puller. The inner surface of the test section was carefully examined with a special optical arrangement and the pouring was repeated in case of cracks and notches.

The ratio of the test section length L to its diameter d was 15. It is known from references [2, 3] that at $Pr \sim 1000$ the stabilization length of a concentration boundary layer ranges from one to two diameters, depending on Re. According to Deissler [3], whose study of the subject is most comprehensive, for a given L/d this effect decreases with increase of Pr at high Re, i.e. the local Nu calculated for the "entrance portion" quickly approaches the value typical for a fully developed flow.

Having taken into account that in the present study $L/d = 15$, one could neglect the effect of the region of stabilization of the concentration boundary layer.

Experiments were carried out on solution of benzoic acid in pure water, glycerine–aqueous solutions (weight per cent of glycerine is 10, 30, 45), aqueous solutions of sucrose (weight per cent of sucrose is 30, 40) and also of naphthalene in methanol–aqueous mixture (volumetric ratio is 1:1). The temperature during the experiments was $20 \pm 0{\cdot}2^{\circ}C$. The concentration of the solute was determined by means of a spectrophotometer СФ-4 (the wave-length for benzoic acid $\lambda = 268$ mμ, for naphthalene $\lambda = 274$ mμ). The duration of a run did not exceed 10 min, this was selected as the least time sufficient for spectrophotometric determination of the amount of solute dissolved. As a result, at the end of the run the inner surface of the

test section of the pipe remained practically as smooth as it was at the beginning. The solubility of benzoic acid and naphthalene in the solvents used was determined experimentally.

The mass-transfer coefficients were calculated by the formula

$$K = \frac{m}{t \,.\, \Delta C_m \,.\, S}. \tag{1}$$

The deviation from the mean value for the mass-transfer coefficients was ± 5 per cent.

The coefficients of molecular diffusion of benzoic acid in water, which are needed for the calculations, are taken from [4]. The coefficients of diffusion of benzoic acid in aqueous solutions of sucrose and glycerine, and naphthalene in the methanol–aqueous mixture, were determined experimentally by studying the mass transfer from rotating discs in the laminar regime of motion. A description of the device is given in reference [4]. The molecular-diffusion coefficients were calculated from the experimental data by the Levich formula [5].

$$K = 0{\cdot}62\, D^{\frac{2}{3}} \nu^{-\frac{1}{6}} \omega^{\frac{1}{2}}. \tag{2}$$

The accuracy of the determination was ± 5 per cent.

In Table 1 the values of molecular-diffusion coefficients of benzoic acid and naphthalene in the solvents used, and the values of solubility are presented. For the density and viscosity of the solutions of sucrose and glycerine tabulated values were used.

The experimental results on mass transfer from the tube wall to a turbulent liquid flow are given in Table 2.

All the theoretical formulae which describe mass transfer in tubes with turbulent flow and high Prandtl numbers [1, 3, 5, 7–9] can be finally reduced to the form

$$Nu \sim \zeta^{0{\cdot}5}\, Re\, Pr^n \tag{3}$$

where ζ is the coefficient of resistance in the pipe which is calculated for a Reynolds number not higher than 10^5 by the Blasius formula. This is equivalent to a dependence of Nu on Re to the power 0·88. As to the effect of Pr, there is no common opinion [6]. Depending upon the law of turbulent heat-transfer coefficient damping in the viscous sublayer selected, $n = 0{\cdot}25$ [3, 5, 7], $n = 0{\cdot}33$ [8, 9] or $n = 0{\cdot}5$ [1]. For this reason the experimental data presented as $Nu/Re \,.\, \zeta^{0{\cdot}5}$ versus Pr (Fig. 3).

As will be seen from the graph, the experimental data lie on a straight line, the slope of

Table 1. The values of the coefficients of molecular diffusion and solubility of the studied systems

System	Solubility $C_{sat} \times 10^6$ (mol/cm^3)	Coefficient of molecular diffusion $D \times 10^6$ (cm^2/s)
Benzoic acid–water	23·7	9·5
Benzoic acid–10 per cent glycerine–water solution	24·5	7·0
Benzoic acid–30 per cent glycerine–water solution	27·9	3·7
Benzoic acid–45 per cent glycerine–water solution	38·2	2·0
Benzoic acid–30 per cent sucrose–water solution	23·9	2·5
Benzoic acid–40 per cent sucrose–water solution	23·8	1·7
Naphthalene–methanol–aqueous mixture (1:1)	6·31	5·5

Table 2. Experimental data on mass transfer from tube wall to turbulent liquid flow

System	Pr	Re	Nu	$Nu/\zeta^{0.5}Re$
Benzoic acid–water	1060	11060	770	0·397
	1060	21740	1300	0·370
	1060	21870	1300	0·369
	1060	22280	1450	0·405
	1060	24080	1450	0·378
	1060	36900	2300	0·413
	1060	37270	2220	0·395
Benzoic acid–10 per cent glycerine–water solution	1830	28940	2230	0·495
	1830	28940	2230	0·495
	1830	29070	2250	0·497
Naphthalene–methanol–aqueous mixture (1:1)	3710	10360	1440	0·785
	3710	13860	1800	0·761
	3710	21600	2740	0·786
	3710	26640	3120	0·745
	3710	30940	3330	0·697
	3710	53290	5810	0·756
	3710	59950	6550	0·769
Benzoic acid–30 per cent glycerine–water solution	6300	40000	6360	1·063
	6300	40000	5300	0·887
Benzoic acid–30 per cent sucrose–aqueous solution	11260	30730	6600	1·390
Benzoic acid–45 per cent water–glycerine solution	21190	16380	4290	1·567
	21190	16380	4610	1·686
Benzoic acid–40 per cent sucrose–aqueous solution	31010	15560	5510	2·107
	31010	16380	5580	2·039
	31010	16380	5580	2·039

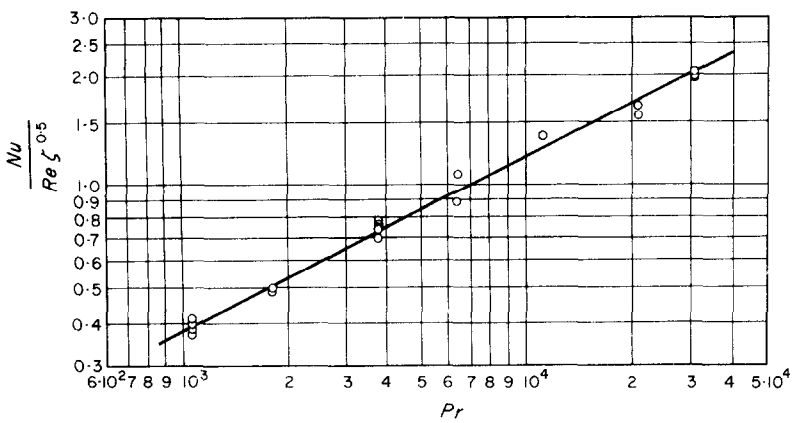

FIG. 3. $Nu/Re \cdot \zeta^{0.5}$ versus Pr.

which calculated by the least squares method is 0·49.

The experimental results of the present work thus allow the conclusion that, in the case of mass transfer from a smooth tube wall to a turbulent liquid flow, $Nu \sim Pr^{0.5}$, which corresponds to the proportionality of the turbulent transfer coefficient in the viscous sublayer to the square of the distance from the wall [1].

It should be noted that in most experimental

studies [10–14] dealing with the rate of solution of solids in liquids, the same dependence of Nu on Pr has been established.

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POSTSCRIPT

After the present paper had been sent to the publisher a report by P. Harriott and R. Hamilton appeared in *Chem. Engng Sci.* **20**, 1073–1078 (1965). The authors have carried out experimental research on mass transfer from a smooth pipe wall to a turbulent liquid flow over the range of Schmidt numbers from 430 to 100000 and obtained the correlation

$$Nu = 0{\cdot}0096\, Re^{0{\cdot}913}\, Sc^{0{\cdot}346}.$$

The discrepancy between the experimental results just mentioned and the data of the present paper is possibly due to differences of frequencies and amplitudes of vibrations of the experimental installations.

Abstract—The paper deals with experimental data on mass transfer from the wall of a smooth tube to a turbulent liquid flow. The data have been obtained for the Prandtl and Reynolds numbers within the range from 1000 to 31000 and from 10000 to 60000, respectively. The results obtained are described by the relation $Nu \sim Pr^{0{\cdot}5}$.

Résumé—L'article décrit les données expérimentales sur le transport de masse à partir de la paroi d'un tube lisse avec un écoulement liquide turbulent. Les expériences ont été faites avec des nombres de Prandtl et de Reynolds compris respectivement entre 1000 et 31000 et entre 10000 et 60000. Les résultats obtenus obéissent à la relation $Nu \sim Pr^{0,5}$.

Zusammenfassung—Die Arbeit behandelt die experimentelle Untersuchung des Stofftransports von der Wand eines glatten Rohres bei turbulenter Flüssigkeitsströmung. Der beobachtete Prandtlbereich erstreckte sich von Pr = 1000–31000 der Reynoldsbereich von Re = 10000–60000. Die Ergebnisse können mit der Beziehung $Nu \sim Pr^{0,5}$ beschrieben werden.